The Study of the Orbital Period of Saturn's Moons for determining the Saturn's mass

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Abstract

This project is about the orbital period of Saturn's Moons, the distance from Saturn's Moons to Saturn and Saturn's mass with Kepler's third law and Newton's law of motion. Which uses trigonometric function for determine the Saturn's mass and compare pixel spacing to angular distance with M27.

Introduction

The Saturn's ring can be observed clearly. This makes it possible to find Saturn's planes for observers on Earth. Based on trigonometry, it calculates the period and distance of the moon as well as the Saturn's mass.

Method

Part 1 : Finding the orbital period of Saturn's Moons.

1. Adjust the lighting of Saturn's picture to see the moons clearly with IRIS and save as JPEG files.
2. Draw ovals of Saturn's rings by using Photoshop and find the major axis length(x) and minor axis length(y).
3. Measure distance from the moon to Saturn along the x-axis(H) and y-axis(W) cause $\theta = \arctan\left(\frac{yW}{xH}\right)$
4. Compare 2 pictures, determined by the angle and the period of time spent to take photos, then find the orbital period from $T = \frac{\Delta t}{\theta_2 - \theta_1} 2\pi$

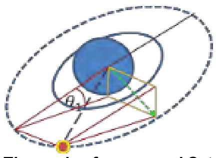

Fig. 1 : The angle of moon and Saturn

Part 2 : Finding the distance from Saturn's Moons to Saturn.

1. Bring M27 for analysis and compare pixel spacing to angular distance.
2. Measure distance from moon to Saturn (pixel) and change to angular distance
3. Find **"d"** by Parallax method

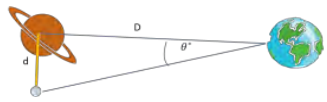

Fig. 2 : The relation between D and d

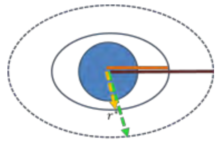

Fig 3 : Lengths that can find on the picture.

4. Find the real distance by

$$\text{Distance from moon to Saturn} = \frac{d \cdot \text{semi major axis length}}{\text{distance from center to } r^*}$$

Part 3 : Finding the Saturn's mass

Use the Orbital period and the distance to measure Saturn's mass by $M = \frac{4\pi^2 a^3}{GT^2}$

Results

Table 1 : The orbital period of Saturn's Moons.

Time	Orbital Period (Days)				
	Enceladus	Tethys	Dione	Rhea	Titan
1	-	1.89	2.85	4.29	13.25
2	1.36	1.88	2.65	4.43	14.36
3	-	1.89	2.85	4.66	20.89
4	-	1.88	2.77	4.50	16.18
5	-	1.88	2.74	4.55	17.00
6	-	1.89	2.66	4.50	13.83
7	-	1.87	2.58	4.79	15.19
Average	**1.36**	**1.88**	**2.73**	**4.53**	**15.81**

Table 2 : The distance from Saturn's Moons to Saturn.

Time	The distance from Saturn's moons to Saturn (Mm)				
	Enceladus	Tethys	Dione	Rhea	Titan
1	-	289.90	347.06	481.16	1144.38
2	239.64	285.14	354.10	517.02	1120.07
3	256.11	276.92	394.90	505.05	1222.88
4	-	275.92	338.48	464.72	1298.66
5	217.10	284.09	374.27	449.08	1061.09
6	-	320.05	400.00	502.47	1095.05
7	-	273.83	387.34	518.91	1216.33
8	-	238.14	366.50	519.43	1139.87
Average	**237.62**	**280.50**	**370.32**	**494.31**	**1162.37**

Table 3 : The Saturn's mass

Saturn's Moons	Orbital Period (second)	Distance from Saturn (m)	Saturn's mass (kg)	Average Saturn's mass
Enceladus	1.18×10^5	2.38×10^8	5.74×10^{26}	**5.15×10^{26}**
Tethys	1.63×10^5	2.81×10^8	4.94×10^{26}	
Dione	2.36×10^5	3.70×10^8	5.40×10^{26}	
Rhea	3.91×10^5	4.94×10^8	4.67×10^{26}	
Titan	13.66×10^5	11.62×10^8	4.98×10^{26}	

Conclusions and Discussion

This project reveals that the orbital period of Enceladus, Tethys, Dione, Rhea and Titan are 1.36 days, 1.88 days, 2.74 days, 4.53 days, and 15.81 days that the margin of error is 0.65%, 0.27%, 0.29%, 0.42% and 0.83% respectively, and distance from Saturn's moons to Saturn is 237.62 Mm, 280.50 Mm, 370.32 Mm, 494.31 Mm and 1162.37 Mm that the margin of error is 0.14%, 4.79%, 1.87%, 6.22% and 4.87% from major axis length respectively. Saturn's mass is 5.15×10^{26} kg, the margin of error is 9.44%. At any rate, the difference of resolution causes the error.

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Reference

Matipon Tangmatitam. (2013). The Handbook of Astronomic Workshop, Chiang Mai: Educational Astronomic Information Service Center.