R22a Scaling of the Distribution Function under the Vlassov-Poisson Equations and the Critical Exponents near the Point of a Marginal Stability

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A model system, described by the set of the Vlassov-Poisson equations under periodical boundary conditions, has been studied numerically near the point of a marginal stability to investigate the critical dynamics for the case of the long-distance Newton body-body interaction potential. The following power laws, typical for a system, undergoing a second-order phase transition, hold in a vicinity of the critical point: (i) $A \propto -\theta^{\beta}$, $\beta = 1.907 \pm 0.006$ for $\theta \leq 0$, where A is the saturated amplitude of the marginally-stable mode; (ii) $\chi \propto \theta^{-\gamma}$ as $\theta \to 0$, $\gamma = \gamma_- = 1.020 \pm 0.008$ for $\theta < 0$, and $\gamma = \gamma_+ = 0.995 \pm 0.020$ for $\theta > 0$, where $\chi = \partial A/\partial F_1$ at $F_1 \to 0$ is the susceptibility to external driving of the strain F_1; (iii) at $\theta = 0$ the system responds to external driving of strain F_1 as $A \propto F^{1/\delta}$, and $\delta = 1.544 \pm 0.002$. $\theta = (\langle v^2 \rangle - \langle v_{cr}^2 \rangle)/\langle v_{cr}^2 \rangle$ is the dimensionless reduced velocity dispersion. Within the error of computation these critical exponents satisfy to the Widom equality, $\gamma = \beta(\delta - 1)$, which is the direct consequence of scaling invariance of the critical dynamics for the Fourier components f_k of the distribution function f at $|\theta| \ll 1$, i.e. $f_k(\lambda^{a_t}t,\ \lambda^{a_v}v,\ \lambda^{a_\theta}\theta,\ \lambda^{a_{A_0}}A_0,\ \lambda^{a_F}F_1) = \lambda f_k(t, v, \theta, A_0, F_1)$. On the contrary to thermodynamics, where critical phenomena exists in a very narrow area of parameters, these critical indexes indicate to a very wide critical area for gravitation. In turn, it means that critical phenomena may determine macroscopic dynamics of a large fraction of systems.